

3.2 | 연습문제 정답

2023. 2. 10 업데이트

01.

$$(a) \frac{dy}{dx} = 8x(2x^2 + 3)(3x + 1)^3 + 9(2x^2 + 3)^2(3x + 1)^2 \\ = (2x^2 + 3)(3x + 1)^2(42x^2 + 8x + 27)$$

$$(b) \frac{dy}{dx} = -(\sec 2x - 1)^{-\frac{3}{2}} \sec 2x \tan 2x$$

$$(c) \frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}}e^{2x} + 2\sqrt[3]{x}e^{2x}$$

$$(d) \frac{dy}{dx} = 2\left(e^{-\frac{2}{x}}\right)\left(\frac{1}{x^2}\right)$$

$$(e) \frac{dy}{dx} = \frac{-3x^2e^{-x^3}}{e^{-x^3}} = -3x^2$$

$$(f) \frac{dy}{dx} = \begin{cases} \frac{3}{3x+3}, & x > -1 \\ \frac{-3}{-3x-3}, & x < -1 \end{cases}$$

$$(g) \frac{dy}{dx} = -\frac{\csc^2 x}{\cot x} = -\frac{1}{\sin x \cos x}$$

$$(h) \frac{dy}{dx} = \frac{1}{2\sqrt{x}} \cos \sqrt{x} - \left[\frac{1}{2\sqrt{x}} \cos \sqrt{x} + \sqrt{x}(-\sin \sqrt{x}) \frac{1}{2\sqrt{x}} \right] = \frac{\sin \sqrt{x}}{2}$$

$$(i) \frac{dy}{dx} = \frac{1}{2\sqrt{x}} \sin \sqrt{x} + \frac{\sqrt{x}}{2\sqrt{x}} \cos \sqrt{x} + \sqrt{x}(-\sin \sqrt{x}) \frac{1}{2\sqrt{x}} = \frac{\cos \sqrt{x}}{2}$$

$$(j) \frac{dy}{dx} = \frac{1}{2}(a^2 + b^2 - 2ab \cos x)^{-\frac{1}{2}}(2ab \sin x) = (a^2 + b^2 - 2ab \cos x)^{-\frac{1}{2}}(ab \sin x)$$

$$(k) \frac{dy}{dx} = 4\left(x + \frac{1}{x}\right)^3 \left(1 - \frac{1}{x^2}\right)$$

$$(l) \frac{dy}{dx} = 2x2^x + \ln 2(x^2)2^x = 2^x x(2 + x \ln 2)$$

$$(m) \frac{dy}{dx} = \frac{x - \sqrt{x^2 + 4} - x \left(1 - \frac{2x}{2\sqrt{x^2 + 4}} \right)}{(x - \sqrt{x^2 + 4})^2} = \frac{x - \sqrt{x^2 + 4} - x + \frac{x^2}{\sqrt{x^2 + 4}}}{(x - \sqrt{x^2 + 4})^2}$$

$$= - \frac{4}{\sqrt{x^2 + 4} (x - \sqrt{x^2 + 4})^2}$$

$$(n) \cos(x+y) \left(1 + \frac{dy}{dx} \right) + \cos(x-y) \left(1 - \frac{dy}{dx} \right) = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(x+y) + \cos(x-y)}{\cos(x-y) - \cos(x+y)} = \frac{\cos x \cos y}{\sin x \sin y}$$

$$(o) \frac{dy}{dx} = \frac{\sinh x}{\cosh x} = \tanh x$$

$$(p) \frac{dy}{dx} = -\frac{1}{3} (2 \cosh 3x) 3 \sinh 3x = -2 \cosh 3x \sinh 3x$$

02. $y' = 2, y'' = -15$

03. $y' = -\frac{4\sqrt{3}}{5}, y'' = -\frac{32}{25}$

04. $y' = 2, y'' = -\frac{2}{3}$

05. $y^{(n)} = (\sqrt{2})^n e^x \sin\left(x + \frac{n}{4}\pi\right), y^{(n)} = (\sqrt{2})^n e^x \cos\left(x - \frac{n}{4}\pi\right)$

06. $y^{(n)} = x^3 (\sin x)^{(n)} + 3nx^2 (\sin x)^{(n-1)} + \binom{n}{2} 6x (\sin x)^{(n-2)} + \binom{n}{3} 6 (\sin x)^{(n-3)}$

07. 1

08. (1) $\frac{du}{dx} = \left(-\frac{1}{2}\right) f'(x)^{-\frac{3}{2}} f''(x)$

$$\Rightarrow \frac{d^2 u}{dx^2} = \left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) f'(x)^{-\frac{5}{2}} f''(x)^2 - \frac{1}{2} f'(x)^{-\frac{3}{2}} f'''(x)$$

$$\begin{aligned} \Rightarrow \frac{1}{u} \frac{d^2 u}{dx^2} &= f'(x)^{\frac{1}{2}} \left(\frac{3}{4}\right) f'(x)^{-\frac{5}{2}} f''(x)^2 - \frac{1}{2} f'(x)^{-1} f'''(x) \\ &= \frac{3}{4} f'(x)^{-2} f''(x)^2 - \frac{1}{2} f'(x)^{-1} f'''(x) \end{aligned}$$

$$(2) \quad \frac{dv}{dx} = f'(x) f'(x)^{-\frac{1}{2}} + f(x) \left(-\frac{1}{2}\right) f'(x)^{-\frac{3}{2}} f''(x)$$

$$= f'(x)^{\frac{1}{2}} - \frac{1}{2} f(x) f'(x)^{-\frac{3}{2}} f''(x)$$

$$\Rightarrow \frac{d^2 v}{dx^2} = \frac{1}{2} f'(x)^{-\frac{1}{2}} f''(x) - \frac{1}{2} f'(x) f'(x)^{-\frac{3}{2}} f''(x)$$

$$+ \frac{3}{4} f(x) f'(x)^{-\frac{5}{2}} f''(x)^2 - \frac{1}{2} f(x) f'(x)^{-\frac{3}{2}} f'''(x)$$

$$\Rightarrow \frac{d^2 v}{dx^2} = \frac{3}{4} f(x) f'(x)^{-\frac{5}{2}} f''(x)^2 - \frac{1}{2} f(x) f'(x)^{-\frac{3}{2}} f'''(x)$$

$$\Rightarrow \frac{1}{v} \frac{d^2 v}{dx^2} = f(x)^{-1} f'(x)^{\frac{1}{2}} \left(\frac{3}{4}\right) f(x) f'(x)^{-\frac{5}{2}} f''(x)^2$$

$$+ f(x)^{-1} f'(x)^{\frac{1}{2}} \left(-\frac{1}{2}\right) f(x) f'(x)^{-\frac{3}{2}} f'''(x)$$

$$= \frac{3}{4} f'(x)^{-2} f''(x)^2 - \frac{1}{2} f'(x)^{-1} f'''(x) \quad \therefore \frac{1}{u} \frac{d^2 u}{dx^2} = \frac{1}{v} \frac{d^2 v}{dx^2}$$

$$09. \quad y' = \frac{2(\sqrt{x+a} + \sqrt{x-a}) \left(\frac{1}{2\sqrt{x+a}} + \frac{1}{2\sqrt{x-a}} \right)}{(\sqrt{x+a} + \sqrt{x-a})^2} = \frac{1}{\sqrt{x^2 - a^2}}$$

$$y'' = \frac{-\frac{1}{2} \frac{2x}{\sqrt{x^2 - a^2}}}{x^2 - a^2} = \frac{-x}{(x^2 - a^2) \sqrt{x^2 - a^2}}$$

$$\therefore y'' + x(y')^3 = \frac{-x}{(x^2 - a^2)\sqrt{x^2 - a^2}} + x\left(\frac{1}{\sqrt{x^2 - a^2}}\right)^3 = 0$$

10. $f'(x) = 2 \cosh x \sinh x, g'(x) = \sinh(x^2)(2x)$

11. $f'(x) = \frac{\cosh x}{\sinh x}, g'(x) = \cosh(\ln x)\left(\frac{1}{x}\right)$

12. $f'(x) = n \sinh nx + n \cosh nx$

13. $\sinh(\ln(\csc\theta + \cot\theta)) = \frac{e^{\ln(\csc\theta + \cot\theta)} - e^{-\ln(\csc\theta + \cot\theta)}}{2}$
 $= \frac{1}{2} \left[\csc\theta + \cot\theta - \frac{1}{\csc\theta + \cot\theta} \right]$ 이고 $\csc^2\theta - \cot^2\theta = -1$ 이므로
 $\frac{1}{2} \left[\csc\theta + \cot\theta - \frac{1}{\csc\theta + \cot\theta} \right] = \frac{1}{2} \left[\csc\theta + \cot\theta - \frac{\csc\theta - \cot\theta}{\csc^2\theta - \cot^2\theta} \right] = \csc\theta.$

14. $y = A \sin kx + B \cos kx \Rightarrow y' = kA \cos kx - kB \sin kx$
 $\Rightarrow y'' = -k^2 A \sin kx - k^2 B \cos kx = -k^2 y$

15. $y = A \sinh kx + B \cosh kx \Rightarrow y' = kA \cosh kx + kB \sinh kx$
 $\Rightarrow y'' = k^2 A \sinh kx + k^2 B \cosh kx = k^2 y$